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Universidade do Algarve

**Matched-Field Processing  
with a Vector Sensor Array**

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Abstract	The main objective of this report is to present a particle velocity-pressure joint data model and a version of Bartlett estimator in order to include the particle velocity in Matched-field inversion techniques. Both data model and estimator will be used in the estimation of bottom parameters such as: the compressional wave speed, the compressional attenuation and the density of sediment, where the simulations results are provided by the TRACE ray tracing model, capable for particle velocity outputs.
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# Abstract

This report presents a vector sensor array (VSA) data model, which merges both the acoustic pressure and the particle velocity components. The signal component of the VSA data model is derived considering a ray tracing model - TRACE, which accounts for particle velocity components. Then a VSA version of Bartlett estimator is developed in order to incorporate the particle velocity in the Matched-Field (MF) inversion techniques, usually done with acoustic pressure. Comparisons between several processors based either in individual particle velocity components and acoustic pressure only or using all the vector sensor outputs, it will be made. The simulations results, for estimating ocean bottom parameters such as: the compressional wave speed  $c_s$ , the compressional attenuation  $\alpha$  and the density  $\rho$  of sediment, are presented considering the TRACE ray tracing model to generate the field replicas. Using the physical output information (acoustic pressure and particle velocity) of TRACE, various simulations cases are shown in order to determine the MF output sensitivity of the parameter variability. Moreover, the influence of reducing the number of element sensors in the bottom parameters estimation is shown.

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# Chapter 1

## Introduction

The vector sensors measure both the acoustic pressure and the three components of particle velocity and are normally configured as vector sensor arrays (VSA). This type of sensor has the ability to provide information in both vertical and azimuthal direction and has the advantage of being able to provide substantially higher directivity with much smaller aperture than an array of traditional scalar (pressure only) hydrophones. The improved spatial filtering capabilities of a VSA, when compared with traditional pressure-only sensor arrays, provide a clear advantage in source localization and related problems. This issue was shown in [1].

To understand how pressure and particle velocity interact and propagate in the ocean waveguide, models that use both acoustic pressure and particle velocity must be used, to demonstrate processing capabilities that exploit the unique characteristics of vector sensors and to validate the VSA data. The TRACE ray tracing model [2], provide different sets of output information, which one can be acoustic pressure and particle velocity.

The classical matched-field processing (MFP) perform a comparison of the full pressure field (amplitude and phase) received at an array of hydrophones with computer generated field replicas, usually by means of a correlation [3]. Here, a MFP based on pressure and particle velocity fields is going to be used for geoacoustic inversion purposes.

In order to approximate the simulations results to the VSA real data collected during the Makai experiment, various simulations cases are presented with similar environmental and geometric scenario of the MakaiEx'05 sea trial, which experiment was well described in [4]. The simulations results show the influence of different numbers of sensors (4, 6, 10 and 20) considering the frequency of 8254 Hz in the sensitivity of the three parameters of the sediment (the compressional wave speed  $c_s$ , the compressional attenuation  $\alpha$  and the density  $\rho$ ), before processing the real data. The objective of this simulations is to take advantage of using synthetic data without noise to estimate the ocean bottom parameters in an ideal case, understand the variability of the MFP output in different situations, and determine the best way to follow when the real data is used.

This report is organized as follows: chapter 2 develops the vector sensor data model based on a ray tracing approach and the theory related to the Bartlett estimator based on particle velocity for generic parameter estimation; chapter 3 makes a short description of canonical scenario and a general description of the TRACE model used; chapter 4 presents the simulations results considering the derived VSA-based Bartlett estimators with all or individual components applied for seabed parameter estimation, and these estimators were tested with several number of element sensors; and finally the chapter 5 concludes this report.

# Chapter 2

## Vector sensor array processing

In order to understand how the vector sensor or how the components of particle velocity influences the parameter estimation, it is important to develop a model and derive a processor which accounts for pressure and particle velocity information.

In this chapter a VSA data model, which merges the acoustic pressure and the particle velocity components is proposed for generic parameter estimation. The signal component of the VSA data model is derived considering a ray tracing model, which accounts for particle velocity. The model used to generate the vector sensor field replicas is the TRACE ray tracing model [2], which was designed to perform two dimensional acoustic ray tracing in ocean waveguide. The TRACE ray tracing model will be generally described in next chapter.

### 2.1 The data model

A vector sensor measure the acoustic pressure and the acoustic particle velocity. Assuming that the propagation channel is a linear time-invariant system,  $p$  is the acoustic pressure and  $v_x$ ,  $v_y$  and  $v_z$  are the three particle velocity components, then the field measured at the vector sensor due to a source signal  $s(t)$  is given by:

$$y_p(t, \Theta_0) = h_p(\Theta_0) * s(t) + n_p(t), \quad (2.1)$$

$$y_{v_x}(t, \Theta_0) = h_{v_x}(\Theta_0) * s(t) + n_{v_x}(t), \quad (2.2)$$

$$y_{v_y}(t, \Theta_0) = h_{v_y}(\Theta_0) * s(t) + n_{v_y}(t), \quad (2.3)$$

$$y_{v_z}(t, \Theta_0) = h_{v_z}(\Theta_0) * s(t) + n_{v_z}(t), \quad (2.4)$$

where  $*$  is the convolution,  $\Theta_0$  is a vector of relevant parameters,  $h_l(\Theta_0)$  is the channel impulse response and  $n_l(t)$  is the additive noise for pressure and the three components of particle velocity,  $l = p, v_x, v_y, v_z$  respectively.

Assuming a narrowband signal, the sensor output at a frequency  $\omega$  for a particular set of channel parameters  $\Theta_0$  can be rewrite as:

$$Y_p(\omega, \Theta_0) = H_p(\omega, \Theta_0)S(\omega) + N_p(\omega), \quad (2.5)$$

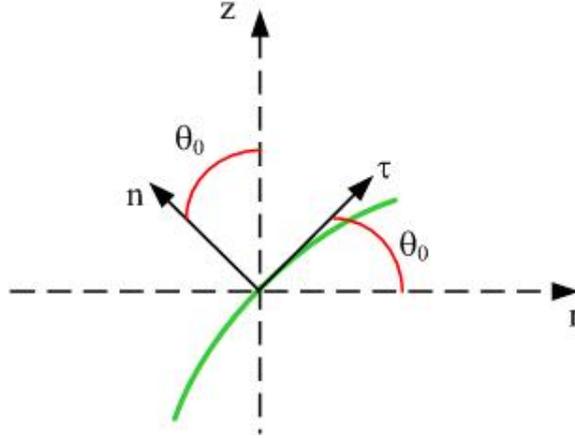


Figure 2.1: Ray trajectory (green line) with ray versors  $\boldsymbol{\tau}$  and  $\mathbf{n}$  onto the horizontal  $r$  and vertical  $z$  axes.

$$Y_{v_x}(\omega, \Theta_0) = H_{v_x}(\omega, \Theta_0)S(\omega) + N_{v_x}(\omega), \quad (2.6)$$

$$Y_{v_y}(\omega, \Theta_0) = H_{v_y}(\omega, \Theta_0)S(\omega) + N_{v_y}(\omega), \quad (2.7)$$

$$Y_{v_z}(\omega, \Theta_0) = H_{v_z}(\omega, \Theta_0)S(\omega) + N_{v_z}(\omega), \quad (2.8)$$

where  $S(\omega)$  is the source spectrum,  $H_l(\omega, \Theta_0)$  is the channel frequency response for pressure and the three components of particle velocity,  $l = p, v_x, v_y, v_z$  respectively, as the same for additive noise.

## 2.2 Particle velocity model formulation

For the vector sensor array (VSA) matched-field inversion, the particle velocity must be modeled.

Let us consider the general geometry of the tangent ( $\boldsymbol{\tau}$ ) and the normal ( $\mathbf{n}$ ) ray versors, at a particular point of the ray trajectory, green line in Fig. 2.1.

The horizontal and vertical particle velocity components ( $v_r, v_z$ ) can be obtained projecting the pressure gradient onto the  $(r, z)$  axes:

$$v_r = -\frac{\partial p}{\partial n} \sin \theta_0 + \frac{\partial p}{\partial s} \cos \theta_0 \quad \text{and} \quad v_z = \frac{\partial p}{\partial n} \cos \theta_0 + \frac{\partial p}{\partial s} \sin \theta_0, \quad (2.9)$$

where  $\theta_0$  is the angle between the  $(r, z)$  axes and the ray versors.

Taking into account that:

$$\boldsymbol{\tau} = [\cos \theta_0, \sin \theta_0] \quad \text{and} \quad \mathbf{n} = [-\sin \theta_0, \cos \theta_0], \quad (2.10)$$

and representing the gradient as:

$$\nabla p = \left[ \frac{\partial p}{\partial n}, \frac{\partial p}{\partial s} \right]. \quad (2.11)$$

Due to the TRACE ray tracing model generate two dimensional components and the VSA has three components, the  $v_x$  and  $v_y$  components are calculated, projecting the horizontal particle velocity in the azimuthal direction of the source ( $\varphi_S$ ), previously estimated. Then:

$$v_x = v_r \cos(\varphi_S) \quad \text{and} \quad v_y = v_r \sin(\varphi_S). \quad (2.12)$$

Using the analytical approximation of the ray pressure as [2, 5]:

$$P(s, n) = P_0(s) \exp \left[ -i\omega \left( \frac{s}{c(s)} + \frac{1}{2} \gamma(s) n^2 \right) \right], \quad (2.13)$$

where  $n$  is the normal distance from the central ray,  $s$  is the arclength along the ray,  $c(s)$  is the sound speed at position  $s$ ,  $P_0(s)$  is an arbitrary constant and  $\gamma(s)$  depends on the functions  $p(s)$  and  $q(s)$  from dynamic ray equations[5].

The derivation of analytical expressions for the pressure gradient components corresponds to:

$$\frac{\partial p}{\partial n} = -i\omega \gamma(s) n p \quad \text{and} \quad \frac{\partial p}{\partial s} = -i \frac{\omega}{c} p. \quad (2.14)$$

Then, the particle velocity components can be written as:

$$\begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = \begin{bmatrix} i\omega \gamma(s) n \sin \theta_0 \cos \varphi_S - i \frac{\omega}{c} \cos \theta_0 \cos \varphi_S \\ i\omega \gamma(s) n \sin \theta_0 \sin \varphi_S - i \frac{\omega}{c} \cos \theta_0 \sin \varphi_S \\ -i\omega \gamma(s) n \cos \theta_0 - i \frac{\omega}{c} \sin \theta_0 \end{bmatrix} p, \quad (2.15)$$

where angle  $\theta_0$  depends on the characteristics of the acoustic channel, including ocean bottom parameters. Assuming that a set environmental parameters characterizing the channel ( $\Theta_0$ ) give rise to an angle  $\theta_0$ , one can write:

$$\mathbf{v}(\Theta_0) = \mathbf{u}(\Theta_0) p, \quad (2.16)$$

where

$$\mathbf{u}(\Theta_0) = \begin{bmatrix} u_x(\Theta_0) \\ u_y(\Theta_0) \\ u_z(\Theta_0) \end{bmatrix} = \begin{bmatrix} i\omega \gamma(s) n \sin \theta_0 \cos \varphi_S - i \frac{\omega}{c} \cos \theta_0 \cos \varphi_S \\ i\omega \gamma(s) n \sin \theta_0 \sin \varphi_S - i \frac{\omega}{c} \cos \theta_0 \sin \varphi_S \\ -i\omega \gamma(s) n \cos \theta_0 - i \frac{\omega}{c} \sin \theta_0 \end{bmatrix}. \quad (2.17)$$

Taking in account (2.16) and (2.17), the particle velocity model can be obtained as:

$$\begin{bmatrix} Y_{v_x}(\omega, \Theta_0) \\ Y_{v_y}(\omega, \Theta_0) \\ Y_{v_z}(\omega, \Theta_0) \end{bmatrix} = \begin{bmatrix} u_x(\Theta_0) H_p(\omega, \Theta_0) \\ u_y(\Theta_0) H_p(\omega, \Theta_0) \\ u_z(\Theta_0) H_p(\omega, \Theta_0) \end{bmatrix} S(\omega) + \begin{bmatrix} N_{v_x}(\omega) \\ N_{v_y}(\omega) \\ N_{v_z}(\omega) \end{bmatrix}. \quad (2.18)$$

## 2.3 Vector sensor array Bartlett estimators

The classical Bartlett estimator is possibly the most widely used estimator in MF parameter identification, usually with the acoustic pressure [3]. This estimator maximizes the output power for a given input signal.

For an array of  $L$  vector sensors, the  $x$  particle velocity component for a given frequency ( $\omega$ ) (omitting the frequency dependency in the following formulas) is given by:

$$\mathbf{Y}_{v_x}(\Theta_0) = [Y_{v_{x1}}(\Theta_0), \dots, Y_{v_{xL}}(\Theta_0)]^T, \quad (2.19)$$

where  $Y_{v_x^i}(\Theta_0)$  is the  $x$  particle velocity component at  $i^{th}$  vector sensor and similar definitions has been adapted for the others particle velocity components  $\mathbf{Y}_{v_y}(\Theta_0)$  and  $\mathbf{Y}_{v_z}(\Theta_0)$ . Then the particle velocity on the VSA output signal can be written as:

$$\mathbf{Y}_v(\Theta_0) = [\mathbf{Y}_{v_x}(\Theta_0), \mathbf{Y}_{v_y}(\Theta_0), \mathbf{Y}_{v_z}(\Theta_0)]^T. \quad (2.20)$$

### 2.3.1 Particle velocity Bartlett estimator

The Bartlett parameter estimate  $\hat{\Theta}_0$  is given as the argument of the maximum of the functional:

$$\max_{\mathbf{e}_v} E \{ \mathbf{e}_v^H(\Theta) \mathbf{Y}_v(\Theta_0) \mathbf{Y}_v^H(\Theta_0) \mathbf{e}_v(\Theta) \} = \max_{\mathbf{e}_v} \mathbf{e}_v^H(\Theta) E \{ \mathbf{Y}_v(\Theta_0) \mathbf{Y}_v^H(\Theta_0) \} \mathbf{e}_v(\Theta), \quad (2.21)$$

where  $^H$  represents the complex transposition conjugation operator,  $\mathbf{e}_v(\Theta)$  is the model predicted data field,  $E \{ \cdot \}$  denotes statistical expectation and  $E \{ \mathbf{Y}_v(\Theta_0) \mathbf{Y}_v^H(\Theta_0) \}$  is the correlation matrix for the particle velocity,  $\mathbf{R}_v(\Theta_0)$ .

Then:

$$\mathbf{R}_v(\Theta_0) = \begin{bmatrix} u_x(\Theta_0) \mathbf{H}_p(\Theta_0) \\ u_y(\Theta_0) \mathbf{H}_p(\Theta_0) \\ u_z(\Theta_0) \mathbf{H}_p(\Theta_0) \end{bmatrix} \begin{bmatrix} u_x(\Theta_0) \mathbf{H}_p(\Theta_0) \\ u_y(\Theta_0) \mathbf{H}_p(\Theta_0) \\ u_z(\Theta_0) \mathbf{H}_p(\Theta_0) \end{bmatrix}^H E \{ S^2 \} + \sigma^2 I, \quad (2.22)$$

where  $\sigma^2 I$  is the noise covariance matrix, assuming that the additive noise is zero-mean spatially white, Gaussian, uncorrelated with the signal and uncorrelated with all components of the vector sensor.

The correlation matrix is usually unknown than an estimated correlation matrix  $\hat{\mathbf{R}}_v$  is used, becoming the Bartlett estimator for the particle velocity outputs as:

$$P_{B,v}(\Theta) = \mathbf{e}_v^H(\Theta) \hat{\mathbf{R}}_v(\Theta_0) \mathbf{e}_v(\Theta), \quad (2.23)$$

where  $\hat{\mathbf{R}}_v$  is defined bellow. The replica vector is proportional to the replica vector for the acoustic pressure as:

$$\mathbf{e}_v(\Theta) = \mathbf{u}(\Theta) \mathbf{e}_p(\Theta). \quad (2.24)$$

Thus, the particle velocity Bartlett estimator is given by:

$$P_{B,v}(\Theta) = |\mathbf{u}(\Theta)^H \cdot \mathbf{u}(\Theta_0)|^2 P_{B,p}(\Theta) \propto |\cos(\delta)|^2 P_{B,p}(\Theta), \quad (2.25)$$

where  $\delta$  is the angle between the replica vector  $\mathbf{u}(\Theta)$  and the data vector  $\mathbf{u}(\Theta_0)$  and

$$P_{B,p}(\Theta) = \mathbf{e}_p^H(\Theta) \hat{\mathbf{R}}_p(\Theta_0) \mathbf{e}_p(\Theta), \quad (2.26)$$

is the Bartlett response when only pressure sensors are considered [6] and  $\hat{\mathbf{R}}_p$  is the estimated correlation matrix for an array of  $L$  sensors of the acoustic pressure only.

Assuming that there is available  $K$  snapshots of data and ergodic process, than the estimated correlation matrix for the acoustic pressure is given by:

$$\hat{\mathbf{R}}_p(\Theta_0) = \frac{1}{K} \sum_{k=1}^K \mathbf{Y}_{k,p}(\Theta_0) \mathbf{Y}_{k,p}^H(\Theta_0). \quad (2.27)$$

A similar definition can be used to estimate the correlation matrix for the particle velocity. Assuming that  $\mathbf{u}(\Theta_0)$  is approximately equal in all receivers because of short length of the array,  $\hat{\mathbf{R}}_v(\Theta_0)$  is given by:

$$\begin{aligned}\hat{\mathbf{R}}_v(\Theta_0) &= \mathbf{u}(\Theta_0) \frac{1}{K} \sum_{k=1}^K \mathbf{Y}_{p,k}(\Theta_0) \cdot \mathbf{Y}_{p,k}^H(\Theta_0) \mathbf{u}^H(\Theta_0) \\ &= \mathbf{u}(\Theta_0) \hat{\mathbf{R}}_p(\Theta_0) \mathbf{u}^H(\Theta_0).\end{aligned}\quad (2.28)$$

One can conclude that the particle velocity Bartlett estimator response is proportional to the pressure Bartlett response by a directivity factor  $|\cos(\delta)|^2$ , given by the inner product  $\mathbf{u}(\Theta)^H \cdot \mathbf{u}(\Theta_0)$ . This directivity factor could provide an improved side lobe reduction or even suppression when compared with the pressure Bartlett response. In the narrowband case, the search is made over the parameter  $\Theta$  and when the maximum is selected an estimation of the parameter  $\Theta_0$  is obtained.

### 2.3.2 Acoustic pressure with particle velocity components

The VSA measures both the acoustic pressure and the particle velocity components. In this section will see the effect of include the acoustic pressure with the particle velocity in Bartlett estimator previously defined.

The VSA output signal is now defined as:

$$\mathbf{Y}_{pv}(\Theta_0) = [\mathbf{Y}_p(\Theta_0), \mathbf{Y}_{v_x}(\Theta_0), \mathbf{Y}_{v_y}(\Theta_0), \mathbf{Y}_{v_z}(\Theta_0)]^T; \quad (2.29)$$

Then the correlation matrix for the VSA data can be written similarly to (2.22) as:

$$\begin{aligned}\mathbf{R}_{pv}(\Theta_0) &= E \{ \mathbf{Y}_{pv}(\omega, \Theta_0) \cdot \mathbf{Y}_{pv}^H(\Theta_0) \} \\ &= \begin{bmatrix} \mathbf{H}_p(\Theta_0) \\ u_x(\Theta_0) \mathbf{H}_p(\Theta_0) \\ u_y(\Theta_0) \mathbf{H}_p(\Theta_0) \\ u_z(\Theta_0) \mathbf{H}_p(\Theta_0) \end{bmatrix} \begin{bmatrix} \mathbf{H}_p(\Theta_0) \\ u_x(\Theta_0) \mathbf{H}_p(\Theta_0) \\ u_y(\Theta_0) \mathbf{H}_p(\Theta_0) \\ u_z(\Theta_0) \mathbf{H}_p(\Theta_0) \end{bmatrix}^H E \{ S^2 \} + \sigma^2 I, \end{aligned}\quad (2.30)$$

and assuming that the additive noise is zero-mean spatially white, Gaussian, uncorrelated with the signal and uncorrelated with all components of the vector sensor.

The correlation matrix is unknown, than an estimated correlation matrix  $\hat{\mathbf{R}}_{pv}(\Theta_0)$  is used and is given by:

$$\hat{\mathbf{R}}_{pv}(\Theta_0) = \begin{bmatrix} 1 \\ \mathbf{u}(\Theta_0) \end{bmatrix} \hat{\mathbf{R}}_p(\Theta_0) \begin{bmatrix} 1 \\ \mathbf{u}(\Theta_0) \end{bmatrix}^H. \quad (2.31)$$

Considering that the VSA replica vector is proportional to the replica for the acoustic pressure as:

$$\mathbf{e}_{pv}(\Theta) = \begin{bmatrix} 1 \\ \mathbf{u}(\Theta) \end{bmatrix} \mathbf{e}_p(\Theta), \quad (2.32)$$

the VSA Bartlett estimator for all components of the VSA is given by:

$$\begin{aligned}
 P_{B,pv}(\Theta) &= \left( \begin{bmatrix} 1 \\ \mathbf{u}(\Theta) \end{bmatrix}^H \cdot \begin{bmatrix} 1 \\ \mathbf{u}(\Theta_0) \end{bmatrix} \right)^2 P_{B,p}(\Theta) \propto |1 + \cos(\delta)|^2 P_{B,p}(\Theta) \\
 &\propto \left| 2 \cos^2 \frac{(\delta)}{2} \right|^2 P_{B,p}(\Theta). \quad (2.33)
 \end{aligned}$$

One can conclude that when the acoustic pressure is included a wider main lobe is obtained (2.33) due to the new directivity factor obtained  $\left| 2 \cos^2 \frac{(\delta)}{2} \right|^2$ , when compared to the estimator with only particle velocity components (2.23). However, when the acoustic pressure is combined with the particle velocity components eliminates also the ambiguities caused by the factor  $|\cos(\delta)|^2$ . Those conclusions were seen for the direction of arrival (DOA) estimation but can be extended for geoacoustic inversion.

# Chapter 3

## Canonical scenario and the TRACE model

### 3.1 Canonical scenario

Traditionally, ocean acoustic signals are measured using hydrophones, which measure the pressure field and are omnidirectional. Recent developments in technology have led to the use of vector sensors in underwater applications, combined in an array of elements, VSA, like the one used during the MakaiEx'05 sea trial from 15 September to 2 October 2005, Fig. 3.1. A 4 element vertical VSA, Fig. 3.1 (b), with 10cm spacing between each element, was used to collect data from towed and fixed acoustic sources. The description of the VSA used and the MakaiEx'05 sea trial was discussed in [4]. The VSA was deployed three times in different situations, being one of the VSA deployment presented here to represent the environmental scenario and the deployment geometry used in simulations. The environmental geometry, with source and receivers location, sound speed profile and the sediment parameters considered in simulations are illustrated in Fig. 3.2. The VSA was deployed with the deepest element at 79.9m depth, in a source-receiver range of 1830 m and source depth of 98 m.

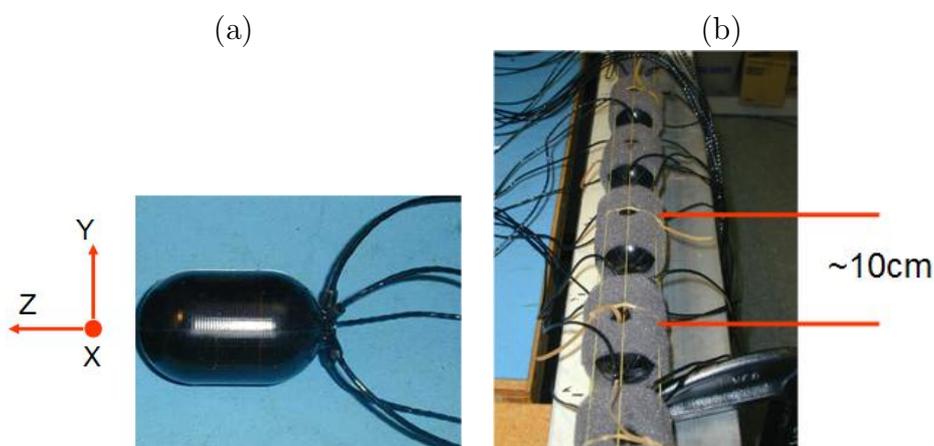


Figure 3.1: Constitution of a single vector sensor with  $x$ ,  $y$  and  $z$  axis orientation (a) and a 5 element vertical VSA with 10 cm spacing view (b).

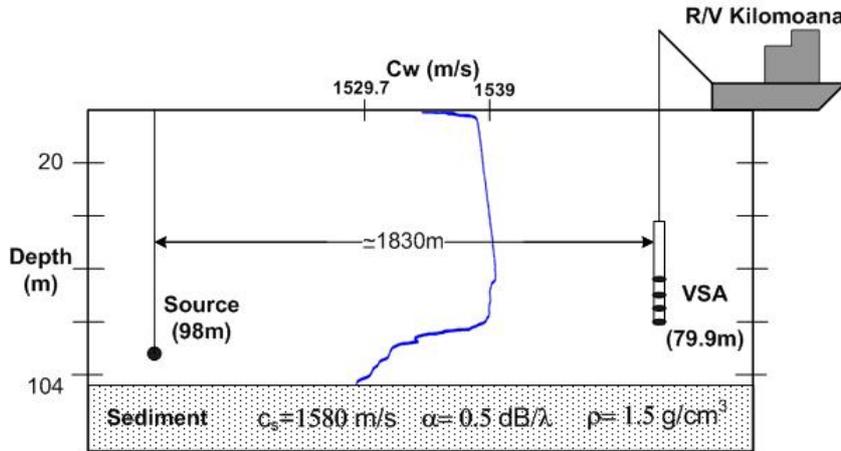


Figure 3.2: Simulation scenario of baseline environment, based on a typical setup encountered during MakaiEx'05, with sound speed profile and sediment parameters values used as “true” values in simulations.

## 3.2 The TRACE propagation model

The TRACE ray tracing model, used to generate the model predicted data field, requires the solution of ray equations to determine the ray coordinates and the dynamic ray equations, that are required for the calculation of ray amplitudes. Both set of equations are described in [2]. To use this model, a WAVFIL (input file) has to be defined corresponding a waveguide environment.

The different blocks are completed with the information of the scene test case, as following:

- *Source Data* – range 1.830 km (source and receivers distance), source range coordinate 0 km, source depth coordinate 98m and source frequency;
- *Altimetry Data* – vacuum over top sediment, sea surface interpolation type 2 points;
- *Sound Speed Data* – sound speed profile shown in Fig. 3.2;
- *Bathymetry Data* – homogeneous bottom with 103m depth and bottom properties: compressional wave speed  $c_s$ , attenuation coefficient  $\alpha$  and density  $\rho$ ;
- *Output Data* – depths of receivers (our case the last one is in depth 79.9m and the others is over this depth with 0.1m spacing) and define the output option: coherent acoustic pressure or coherent particle velocity.

The output option defines the type of calculation to be performed by the model and the type of information which is going to be written, our case: acoustic pressure, horizontal and vertical particle velocity components.

In order to show the advantage of vector sensors in inverse problems using the Bartlett estimator obtained in previous section, three different forms to compare data and replica will be tested:

1. First, the data  $\mathbf{Y}$  and replica  $\mathbf{e}$  in (2.26) are considered as acoustic pressure only, named as  $p$ -only estimator;
2. Second, considering the particle velocity estimator in (2.23) with the components of particle velocity individually - named  $v_r$  for horizontal components and  $v_z$  for vertical component Bartlett estimators;
3. Finally, the VSA estimator - named Bartlett VSA (p + v) estimator, considering a VSA of  $M$  elements. The data and replica vectors are obtained in a long vector approach by:

$$[p_1, \dots, p_M, v_{r_1}, \dots, v_{r_M}, v_{z_1}, \dots, v_{z_M}]^T; \quad (3.1)$$

where  $p_m$  is the acoustic pressure,  $v_{r_m}$  is the horizontal and  $v_{z_m}$  the vertical components of the particle velocity at the  $m^{th}$  sensor.

# Chapter 4

## Simulation results

This chapter presents the simulation results considering different number of vector sensors, in order to understand the influence of the three bottom parameters (sediment compressional speed  $c_s$ , compressional attenuation  $\alpha$  and density  $\rho$ ) in the acoustic field, for the test scenario presented in Fig. 3.2. The frequency used in this simulation is 8254 Hz, corresponding to the first frequency tone of the real VSA data of MakaiEx'05. The TRACE model has three different outputs: the acoustic pressure ( $p$ ), the horizontal particle velocity component ( $v_r$ ) and the vertical particle velocity component ( $v_z$ ). These outputs are useful to see the sensibility of the three bottom parameters in the inversion of acoustic field and what happens when the number of vector sensors is reduced.

MF based inversion technique was performed to compare the acoustic field (amplitude and phase) of synthetic data (with the following bottom properties:  $c_s = 1580$  m/s,  $\alpha = 0.5$  dB/ $\lambda$  and  $\rho = 1.5$  g/cm<sup>3</sup>, used here as “true” values in simulations), received at an array of sensors with model generated field replicas, by means of a correlation process here referred to a Bartlett processor, with a bounded range of possible values for each parameter. The MF was applied to compare the acoustic pressure, the horizontal and vertical components of particle velocity individually (single Bartlett estimator,  $p$ ,  $v_r$  and  $v_z$ ) or combined (VSA (p + v) Bartlett estimator) as seen in section 2.3, without noise.

The estimation performance of the acoustic pressure only (green line), the horizontal particle velocity (cyan line), the vertical particle velocity (blue line) and the VSA Bartlett (red line) estimators are presented in Fig. 4.1, considering 20 element sensors, left panel, and 10 element sensors, right panel, for sediment compressional speed, density and compressional attenuation. From the analysis of the simulation results in Fig. 4.1, some conclusions can be drawn:

1. The field has less sensitivity to the compressional attenuation and has the higher sensitivity to the sediment compressional speed, as expected;
2. It will be difficult to estimate the attenuation and the density, but perhaps the density presents best results than the attenuation;
3. The results for pressure and horizontal particle velocity component are coincident (green and cyan lines) with a large main lobe, since those components mostly depend on low-order modes, thus they depend on rays that have little or no interaction with the bottom;
4. When the number of sensors is reduced to 10, right panel of Fig. 4.1, the estimation results have similar main lobe for compressional speed but with a better resolution for density than with 20 element sensors, exhibiting an uncertainty estimation result for  $\alpha$  parameter;

5. The most important conclusion is that the vertical component (blue line) has the higher sensitivity to bottom structure than the other components, including the VSA (p+v) estimator. This component is influenced by high-order modes with high interaction with the bottom due to their grazing angles. This can be seen by the narrower main lobe in Fig. 4.1 (a) and (b) for compressional sediment speed estimation and even for density, this component presents the best result, Fig. 4.1 (d).

Since the field has less sensitivity to the compressional attenuation, Fig. 4.2 shows only the simulation results for the parameters  $c_s$  and  $\rho$ , considering 6 element sensors (left panel) and 4 element sensors (right panel), with fixed value of the attenuation,  $\alpha = 0.5$  dB/ $\lambda$ . The reduction of the number of the element sensors to 4 is to understand the influence of the field in the realistic case of data acquired by the 4 element VSA during the MakaiEx'05.

The conclusions obtained from the analysis of the Fig. 4.2 confirm those obtained from Fig. 4.1, presented previously. The vertical particle velocity component estimator has the higher sensitivity to both parameters than the others estimators and even for density, a parameter which can be difficult to obtain with pressure only estimator, such component has a good resolution.

Generally, some conclusions can be drawn when the results obtained with different number of sensors are compared. First, the simulation results show that when  $\alpha$  and  $\rho$  are fixed, the parameter  $c_s$  has a narrow main lobe with the maximum well defined, even for the case with 4 element sensors. Second, the parameters  $\alpha$  and  $\rho$  will be difficult to estimate because of wider main lobes obtained. The uncertainty of estimation increases when the number of sensors has been reduced, but perhaps density can be obtained with good resolution when the vertical particle velocity estimator is used. Comparing all results, the VSA (red line) present similar results to the vertical component but presents a wider main lobe for compressional speed. As can be seen in Fig. 4.2, the vertical component (blue line) has the higher sensitivity to bottom structure than the other components. These simulation results show that when the vertical particle velocity component is included in MF inversion it can significantly increase the resolution of bottom parameters estimation and the compressional speed is a parameter which can be estimated with higher accuracy.

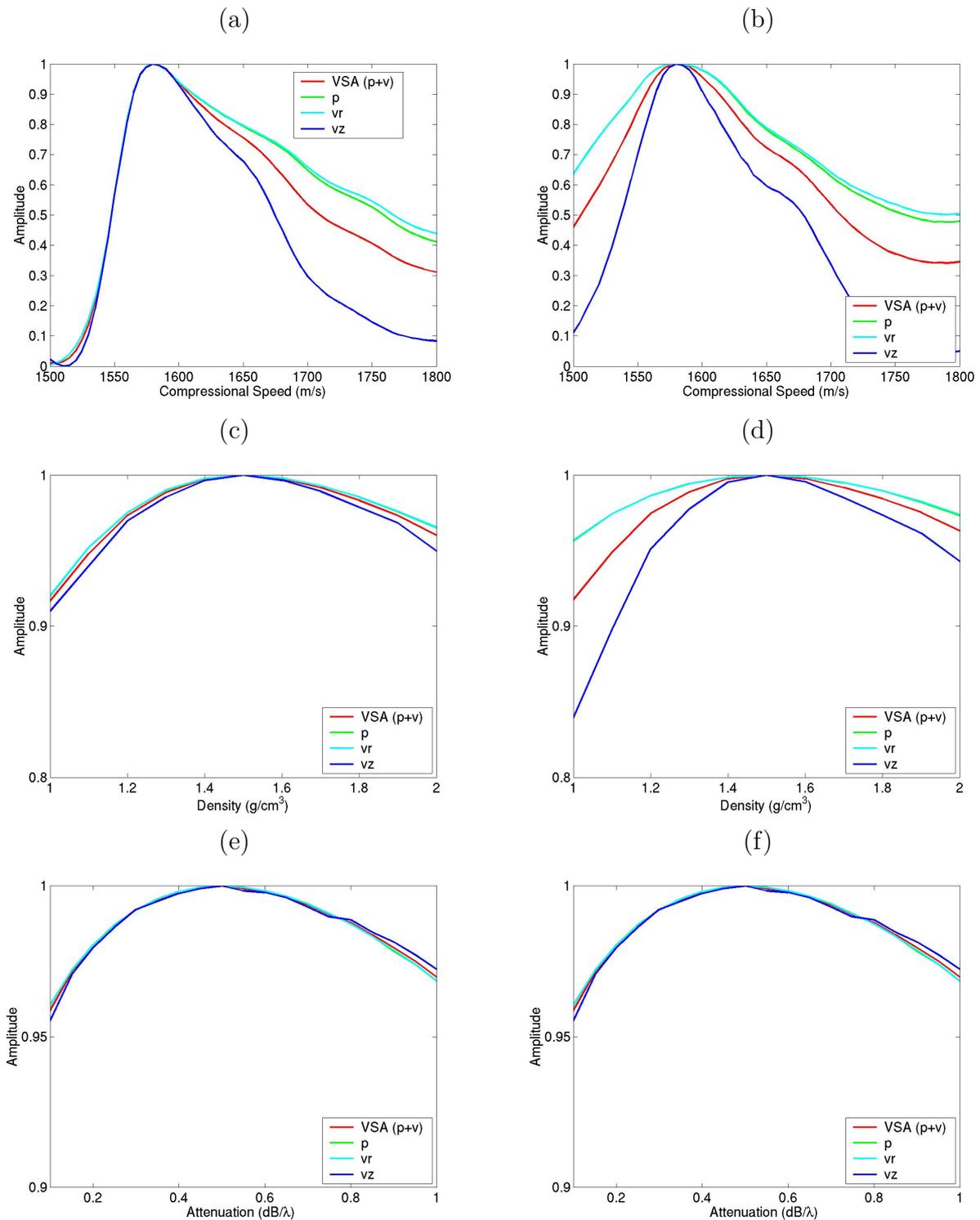


Figure 4.1: Simulations results at frequency 8254 Hz with 20 element sensors (left panel) and 10 element sensors (right panel) for: compressional speed (a) and (b); density (c) and (d) and attenuation (e) and (f).

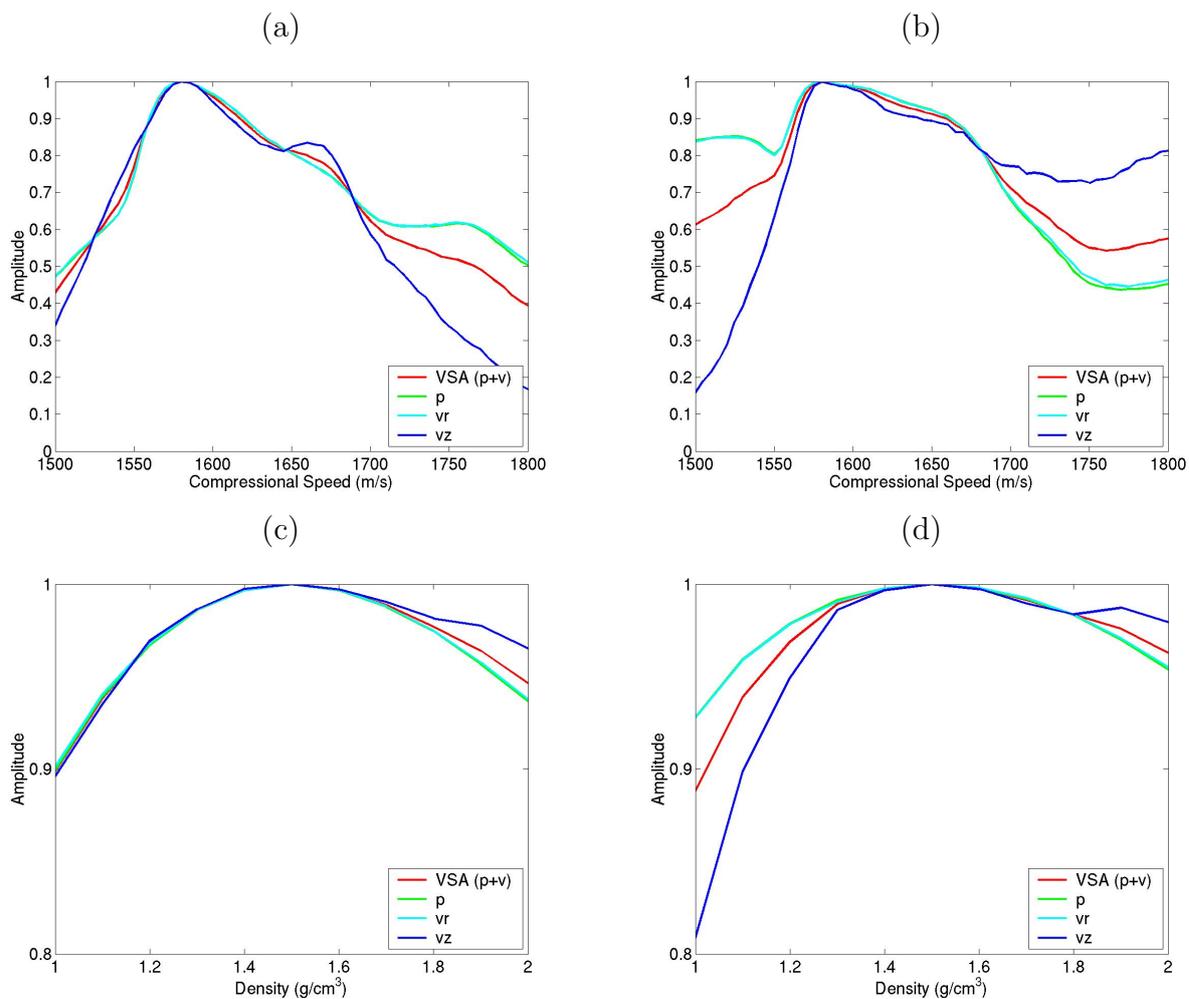


Figure 4.2: Simulations results at frequency 8254Hz with 6 element sensors (left panel) and 4 element sensors (right panel) for: compressional speed (a) and (b), and density (c) and (d).

# Chapter 5

## Conclusion

The present report presented a data model for generic parameter estimation which accounts for particle velocity components, beyond the acoustic pressure. Moreover, it was shown how the conventional Bartlett processor can be adapted in order to incorporate particle velocity outputs. Simulations with the TRACE ray tracing model were performed considering the VSA Bartlett estimator with all or individual components and compared with the pressure only Bartlett estimator. The simulation results were tested with several number of element sensors, concluding that when the number of element sensors is reduced, the VSA Bartlett estimator is able to resolve sediment compressional speed and has sufficient resolution even with a 4 elements VSA. It was shown too that the vertical particle velocity component provided the best estimation resolution for sediment compressional speed and even for attenuation and density, parameters with difficult estimation when pressure only output is considered, presents good resolution. The particle velocity information enhances the ocean bottom parameters estimation, contributing to a better resolution of these parameters.

Future work should be oriented to the study of the following issues:

- the sensitivity of Bartlett VSA estimator to the contamination of data with noise;
- providing MF inversion technique considering others Bartlett estimators with particle velocity components;
- the successful processing of experimental VSA data;
- the broadband processing of VSA data.

# Bibliography

- [1] P. Santos, P. Felisberto, and P. Hursky, “Source localization with vector sensor array during the makai experiment,” in *Proceedings of 2nd International Conference and Exhibition on Underwater Acoustic Measurements: Technologies and Results*, Heraklion, Greece, June 25–29 2007.
- [2] O. C. Rodríguez, *The TRACE and TRACEO ray tracing programs*, <http://www.siplab.fct.ualg.pt/models.shtml>, 2008.
- [3] A. Tolstoy, *Matched Field Processing for Underwater Acoustics*. Singapore: World Scientific, 1993.
- [4] P. Santos, *Vector Sensor Array Data Report Makai Ex 2005*, internal rep 02/08 – siplab/cintal ed., CINTAL – Centro Tecnológico do Algarve, Universidade do Algarve, Campus de Gambelas, 8005-139 Faro, Portugal, March 2008.
- [5] F. B. Jensen, W. A. Kuperman, M. B. Porter, and H. Schmidt, *Computational Ocean Acoustics*. AIP Series in Modern Acoustic and Signal Processing, New York, 1994.
- [6] H. Krim and M. Viberg, “Two decades of array signal processing research,” *IEEE Signal processing magazine*, pp. 67–94, July 1996.